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Inflation and Costs of Price Adjustment

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1. INTRODUCTION

How much? Companies that sell through catalogs run into a pricing problem. Fast rising prices threaten to make catalog quotes out of date. Montgomery Ward & Co. which bought cautiously for its winter catalog, says it is “keeping its fingers crossed” that prices won’t surge much in the near future. An official of Oak Supply & Furniture Co., Chicago, complains that the company’s catalogs have been obsolete in terms of prices before they got into customers’ hands the past two years. Basco Inc., Cherry Hill, N.J. mailed its latest catalog just a few weeks ago but already is reviewing some prices.

J. C. Penney Co. says it stands behind prices during the approximately seven-month life of its semiannual catalogs. Like other companies, it gets guarantees from many vendors that they won’t raise their prices for specified periods. But more catalog sellers tell customers prices are subject to change. Jewelcor Inc. puts such a warning on its catalog’s jewelry pages.

Some companies, such as Basco, consider putting out catalogs more frequently to keep up with prices. For the last two years Sears Roebuck has issued some 20 special catalogs twice rather than once a year. (Wall Street Journal—10/31/74).

As the above quotation indicates, price adjustments are not costless. There are real costs associated with the transmission of price information to the consumers and with the decision process itself. Price changes may be required either because of structural shifts in demand or due to a change in the general price level. The magnitude of the adjustment costs depends critically on whether these changes are anticipated. In principle, if inflation were perfectly anticipated, costs of adjustment could presumably be avoided by a pre-announced formula for price changes. For this reason, the recent literature on monopolistic price adjustment (Barro [2] and Bewley [3]), has focused on the choice of price policies under conditions of random changes in demand or costs. These studies have ignored, however, the role of inflationary expectations in the formation of these policies. The purpose of this paper is to evaluate these effects.

We consider a monopolistic firm that produces a non-storable product whose demand depends on its price relative to the price of rival commodities, considered as an aggregate. The firm expects the aggregate price level and its costs of production to increase at a certain given rate. In the absence of adjustment costs the optimal policy would be to increase its own price continuously at the same rate. We assume, however, that a fixed real charge is associated with each price change. Consequently, the optimal policy is characterized by a sequence of finite intervals during which nominal price is held constant, followed by discrete
price adjustments. Our analysis focuses on the effect of the expected rate of inflation on the frequency and the magnitude of these price changes.

For reasons of simplicity, the model is purely deterministic. Specifically, the aggregate rate of inflation is perfectly anticipated. Furthermore, the real costs of price adjustment are assumed to be independent of the expected rate of inflation. Nevertheless, they may be interpreted as reflecting the costs which are required to eliminate the uncertainty on behalf of consumers concerning changes in relative (in contrast to aggregate) prices.

The main results of the paper can be briefly summarized. In terms of real prices, the firm is shown to follow a policy whereby the nominal price is fixed over intervals of constant duration, with proportionately fixed adjustments between periods. The real price thus fluctuates between two fixed bounds, decreasing continuously over each interval. Formally, identical results were obtained for models in which costs of adjustment are associated with quantities of inputs or outputs (Scarf [5] and Srinivasan [6]).

The focus of this paper is on the effects of changes in various parameters on the optimal policy:

(1) An increase in the rate of inflation is shown to increase the initial price and to decrease the terminal price in each period, thereby increasing the magnitude of each price change. However, its effect on the frequency of price changes is shown to be ambiguous. One expects intuitively that an increase in the rate of inflation will lead to an increase in the frequency of price changes, but we provide a counter example to this conjecture. A sufficient condition for this outcome is that the effect of a nominal price increase on real profits be non-decreasing as the real price decreases.

(2) An increase in the level of real adjustment costs leads to a reduction in the frequency of price changes and consequently to larger adjustments in prices.

(3) An increase in the real rate of interest decreases both the initial and the terminal real prices in each interval. The effect on the frequency of price changes is generally ambiguous. This change is shown to depend on a condition analogous to risk-dominance in the theory of choice under uncertainty.

(4) The imposition of a specific excise-tax on the firm’s output is shown to increase both the initial and the terminal real prices in each period. Again, the effect on the frequency of price changes is shown to depend on a condition identical to risk-dominance.

We also discuss briefly the welfare implications of the problem. In the presence of costs of price adjustment, the socially optimum policy, defined in terms of maximizing consumer’s surplus, also calls for an $(s, S)$ price adjustment policy, although in a different range than the one chosen by the monopoly. This would provide an interesting measure for the “ costs of inflation ” (Bailey [1]).

The plan of this paper is as follows. Section 2 presents the model and derives the optimal policy. A formal proof of the recursive solution is given in an Appendix. Sections 3 and 4 derive and discuss the effects of changes in the rate of inflation, Section 5 the effects of changes in the costs of adjustment and Section 6 the effects of changes in the real rate of interest. Section 7 analyses the effects of taxes on the firm’s policy. Section 8 provides some numerical examples which show that conditions laid out in previous sections to ensure unambiguous results for the effects of some parameter changes are not redundant. Section 9 discusses the welfare implications.

2. A MODEL OF PRICE ADJUSTMENT

Notation

$p_t =$ nominal price charged by the firm at time $t$.

$g =$ rate of inflation (constant).

$\bar{p}_t = e^{gt} =$ general price level at time $t$ (by normalization $\bar{p}_0 = 1$).

$z_t = p_t/\bar{p}_t =$ real price charged by the firm at time $t$. 
\[ q_t = f(z_t) = \text{quantity demanded of the firm's output at time } t \ (f \text{ invariant over time).} \]
\[ c(q_t) = \text{real unit costs of production at time } t \ (c \text{ invariant over time).} \]
\[ F(z_t) = \left[ z_t - c(f(z_t)) \right] f(z_t) = \text{real profits at time } t. \]
\[ \beta = \text{real costs of nominal price adjustment (} \beta \geq 0, \text{ constant).} \]
\[ r = \text{real rate of interest (} r > 0, \text{ constant).} \]
\[ V_0 = \text{present discounted value of real profits at time } 0. \]

Suppose that at time \( t_0 = 0 \) the firm plans to adjust its nominal price at the points of time \( 0 \leq t_1 < t_2 < t_3 < \ldots < t_t < t_{t+1} < \ldots \). Denote the nominally fixed price in the interval \([t_t, t_{t+1})\) by \( p_t \). Then \( p_t e^{-\beta t} \) is the real price at any \( t \) in this interval. Accordingly, total real profits of the firm during this period, including the costs of price adjustment at time \( t_{t+1} \), discounted to time 0, are given by

\[
\int_{t_t}^{t_{t+1}} F(p_t e^{\beta t}) e^{-rt} dt - \beta e^{-rt_{t+1}}. \tag{1}
\]

Summing (1) over \( t \),

\[
V_0 = \sum_{t = 0}^{\infty} \left[ \int_{t_t}^{t_{t+1}} F(p_t e^{\beta t}) e^{-rt} dt - \beta e^{-rt_{t+1}} \right], \tag{2}
\]

where the initial price, \( p_0 \), is assumed to be given, and \( t_0 = 0 \).

The objective of the firm is to choose the sequences \( \{t_t\} \) and \( \{p_t\}, t = 1, 2, \ldots \), that maximize \( V_0 \).

We assume that \( F() \) is differentiable almost everywhere, strictly quasi-concave, that there exists a number \( s^* > 0 \) such that \( F(s^*) > 0 \), and at any \( z \) for which \( F'(z) \) exists

\[
F'(z) \equiv 0 \quad \text{as} \quad z \equiv s^*. \tag{3}
\]

Thus, \( F(z) \) attains a unique maximum at \( s^* \). Further assumptions are required in order to insure that \( V_0 \geq 0 \) at the optimum, i.e. that the firm makes non-negative profits. Specifically, the adjustment costs \( \beta \) should be small relative to \( F(s^*) \).

Assuming that an interior maximum exists, the first-order conditions are:

\[
\frac{\partial V_0}{\partial t_t} = [-F(p_{t-1} e^{-\beta t}) + F(p_{t-1} e^{-\beta t}) + \beta r] e^{-rt} = 0, \quad \tau = 1, 2, \ldots \tag{4}
\]
\[
\frac{\partial V_0}{\partial p_t} = \int_{t_t}^{t_{t+1}} F(p_t e^{-\beta t}) e^{-(\tau + s) dt} = 0, \quad \tau = 1, 2, \ldots \tag{5}
\]

Examining these conditions, it is immediately seen from (5) that when \( g = 0 \), there will be a unique optimal price \( p^* \), such that \( F'(p^*) = 0 \), which holds for all \( \tau \). Consequently, \( \partial V_0 / \partial t_t > 0 \) for any \( \tau \), which means that it is never optimal to change price. It can similarly be seen that if \( \beta = 0 \), the nominal price will change continuously so as to keep the real price constant. The subsequent analysis will thus focus on the non-trivial case \( g \neq 0 \) and \( \beta > 0 \).

It is proved in the Appendix that for any initial price \( p_0 \), a solution to the system (4)-(5) must have a periodic (or recursive) form:

\[
p_t = p_{t-1} e^{\beta t} \quad \text{and} \quad t_{t+1} = t_t + \varepsilon, \quad \tau = 1, 2, \ldots \tag{6}
\]

where \( \varepsilon > 0 \) is a constant. This property follows directly from the independence of the real optimal policy (after the first price change) evaluated at any \( \tau \) of initial conditions. Due to the recursive nature of the solution, the real price in each period, \( p_t \), is seen to move between
two fixed values \((s, S)\), where \(S = se^{\alpha t}\). Changing variables by the transformation 
\(z = p_r e^{-\alpha t}\), conditions (4)-(5) can be expressed in terms of real prices \((z)\) instead of time \((t)\):

\[
F(s) - F(S) + r\beta = 0 \quad \text{...}(4')
\]

\[
\int_s^S F'(z)z^{r/\alpha}dz = 0. \quad \text{...}(5')
\]

Conditions (4')-(5') are two equations to determine the bounds \((s, S)\) on the real price movement.

The value of discounted real profits at the time of the first price change, \(V_1\), is thus given by\(^3\)

\[
V_1 = \frac{1}{1 - e^{-re}} \left[ \int_0^\varepsilon F(p_1 e^{-\alpha t}) e^{-rt} dt - \beta \right]. \quad \text{...(7)}
\]

Using the same transformation as above, (7) can also be expressed in terms of \(s\) and \(S\),

\[
V_1 = \frac{1}{g(S^{r/\alpha} - s^{r/\alpha})} \left[ \int_s^S F(z)z^{(r/\alpha)-1} dz - \beta g S^{r/\alpha} \right]. \quad \text{...(8)}
\]

Differentiating (8) partially w.r.t. \(S\) and \(s\) and equating to zero, we obtain the first-order conditions

\[
F(S) - rV_1 - r\beta = 0 \quad \text{...(9)}
\]

\[-rV_1 + F(s) = 0, \quad \text{...(10)}
\]

which are equivalent to (4') and (5'), as can be seen by integrating the latter by parts. We also find that at any point \((s, S)\) which satisfies conditions (9)-(10),

\[
\frac{\partial^2 V_1}{\partial s^2} = -F'(S)S^{(r/\alpha)-1} < 0, \quad \frac{\partial^2 V_1}{\partial S^2} = \frac{F'(S)S^{(r/\alpha)-1}}{g(S^{r/\alpha} - s^{r/\alpha})} < 0, \quad \frac{\partial^2 V_1}{\partial S \partial s} = 0, \quad \text{...(11)}
\]

where by (3) and (5'), \(F'(S)<0\) and \(F'(s)>0\). Thus, at any stationary point, the second-order conditions are satisfied. This implies that the solution to (9)-(10) is unique.\(^4\) Note also that if there exists a solution to (9)-(10) with \(F(s) > 0\) then, in view of (10), \(V_1 > 0\) at the optimum. Conversely, any solution to (9)-(10) which entails \(F(s) < 0\) cannot be globally optimal.

The interpretation of these equations and the properties of the optimal plan are straightforward. The nominal price is held fixed over an interval \(e\). The real price drifts continuously from the initial level \(S\) to the level \(s\) at the end of the period, at which point a jump occurs and the real price is set again at \(S\). This pattern repeats itself over time. The gain from postponing a price change are the profits just prior to the change, \(F(s)\), and the interest saved on the adjustment costs, \(r\beta\). The loss from such postponement are the profits just after the change, \(F(S)\). Condition (4') states that at the optimum these gains and losses should be equal. Equation (5') states that the nominal price should be set at such a level that the marginal profits due to the change in real prices will average to zero. In view of (3), we have from (5') that \(s < s^* < S\), i.e. the firm operates initially with negative marginal profits, and with positive marginal profits towards the end of each period (Figure 1). Clearly, as implied by (4')-(5'), in the absence of adjustment costs \((\beta = 0)\), the monopoly would continuously operate at the maximum profits point \(s^*\).

Let us make two additional observations. First, due to discounting there is an asymmetry in the solution. The level of real profits after a price increase exceeds by \(r\beta\) the level of real profits just before the price change. Therefore \(s^*\) is in general not the simple mean of \(s\) and \(S\), even if the profit function is symmetric (e.g. quadratic).

Second, equation (5') can be interpreted as stating that if real prices are distributed randomly with a power density function \(z^{r/\alpha}\), expected marginal profits should be equal to
zero. We shall find this analogy with uncertainty useful in the interpretation of the comparative-statics results.

We now turn to an analysis of the dependence of the optimal solution on the parameters \( g, r \) and \( \beta \). There are two aspects of the optimal policy which are of interest: the relative magnitude of price changes, \( S/s = e^{\varepsilon t} \), and the frequency of price changes, i.e. \( \varepsilon = (\ln S - \ln s)/g \).

3. CHANGES IN THE RATE OF INFLATION

Differentiating the system (4′)-(5′) totally with respect to \( g \), we obtain:

\[
\frac{dS}{dg} = \frac{rF'(s)}{g^2\Delta} \int_s^S F'(z)z^{r/\sigma} \ln zdz \quad \ldots(12)
\]

\[
\frac{ds}{dg} = \frac{rF'(S)}{g^2\Delta} \int_s^S F'(z)z^{r/\sigma} \ln zdz \quad \ldots(13)
\]

where

\[
\Delta \equiv F'(S)F'(S)(S^{r/\sigma} - s^{r/\sigma}). \quad \ldots(14)
\]

Since \( F'(s) > 0 \) and \( F'(S) < 0 \), it follows that \( \Delta < 0 \). We now have the following proposition:

**Proposition 1.** \( dS/dg > 0, \; ds/dg < 0 \).

**Proof.** We shall prove that when (5′) is satisfied then

\[
\int_s^S F'(z)z^{r/\sigma} \ln zdz < 0.
\]

Let \( B(z) \equiv \int_s^z F'(x)x^{r/\sigma}dx \). By (5′), \( B(s) = B(S) = 0 \). Also, by (3), \( B(z) > 0 \) for all \( s < z < S \). Integrating by parts

\[
\int_s^S F'(z)z^{r/\sigma} \ln zdz = \int_s^S B'(z) \ln zdz
\]

\[
= B(z) \ln z \bigg|_s^S - \int_s^S \frac{B(z)}{z}dz = -\int_s^S \frac{B(z)}{z}dz < 0. \quad \ldots(15)
\]
Now, by (12), (13) and (15),
\[ -\operatorname{sgn} \frac{dS}{dg} = \operatorname{sgn} \frac{ds}{dg} = \operatorname{sgn} \int_s^S F'(z)z^{r/\theta \ln z}dz < 0. \quad \cdots (16) \]

Proposition 1 establishes that a higher rate of inflation leads to larger nominal price adjustments in each period. This can be seen by observing that, by (6), \( S/s = P_t/P_{t-1} = e^{\theta z} \) for any \( r \). It should be emphasized that this result holds independently of whether the frequency of price changes increases or decreases with the rate of inflation.

Somewhat surprisingly, the effect on \( \varepsilon \) of an increase in the rate of inflation is, in general, ambiguous. It is quite easy to identify the source of this ambiguity. An increase in the rate of inflation has two opposite effects on the net gain from postponement of a price change. On the one hand, it decreases the terminal real price, \( P_1 e^{-\theta z} \), and hence the benefit from postponement, which is the level of real profits just prior to the price change. On the other hand, the costs of postponing a price change, \( rV \), also decrease. This is seen from (10) and Proposition 1,
\[ \frac{\partial V_1}{\partial g} = \frac{F'(s)F'(S)}{g^2 \Delta} \int_s^S F'(z)z^{r/\theta \ln z}dz < 0. \quad \cdots (17) \]

Obviously \( \partial V_1/\partial g = dV_1/dg \) at the optimum. As one would expect, in the presence of adjustment costs, an increase in the rate of inflation reduces the monopoly's real profits.

In order to establish the effect of a change in the rate of inflation on \( \varepsilon \), one has to impose a restriction on the profit function. The condition is that the effect on real profits of a change in the (fixed) nominal price (i.e. \( F'(p_t e^{-\theta z})e^{-\theta z} \)) be a non-decreasing function of time. We state this formally in terms of changes in real prices.\(^5\)

**Monotonicity (M).** \( F'(z)z \) is non-increasing in \( z \).

We now have the following:

**Proposition 2.** Under Condition (M), \( \varepsilon dg/dg < 0 \).

**Proof.** By definition,
\[ \frac{de}{dg} = \frac{1}{g} \left[ 1 \frac{dS}{dg} - 1 \frac{ds}{dg} - \varepsilon \right] = \frac{1}{g} \left[ \frac{d \ln S}{dg} - \frac{d \ln s}{dg} - (\ln S - \ln s) \right]. \quad \cdots (18) \]

Substituting from (12), (13) and (14)
\[ \frac{de}{dg} = \frac{1}{g^2} \left[ \frac{r}{\Delta} \left( \frac{F'(s)}{S} - \frac{F'(S)}{s} \right) \right] \int_s^S F'(z)z^{r/\theta \ln z}dz - (\ln S - \ln s) \]
\[ = \frac{r}{g^3 \Delta} \int_s^S G(z)z^{r/\theta \ln z}dz, \quad \cdots (19) \]
where, upon substitution from (9),\(^6\)
\[ G(z) \equiv \left( \frac{F'(s)}{S} - \frac{F'(S)}{s} \right) F'(z)(\ln z - \ln S) - F'(S)(\ln S - \ln s). \quad \cdots (20) \]

Define a function \( H(z) \):
\[ H(z) \equiv \frac{F'(s)}{S} F'(z)(\ln s - \ln S). \quad \cdots (21) \]
It is easy to verify that $G(s) = H(s)$ and $G(S) = H(S)$. Further, we want to show that $G(z) - H(z) > 0$ for all $s < z < S$. It is easy to see that $G(s^*) - H(s^*) > 0$. Consider the case when $s < z < s^*$. From (19) and (20),

$$G(z) - H(z) = \frac{F'(s)}{S} F'(z)z(\ln z - \ln s) - \frac{F'(S)}{S} [F'(s)s(\ln S - \ln s) - F'(z)z(\ln S - \ln z)].$$

...(22)

The first term on the R.H.S. of (16) is positive since $F'(z) > 0$ for $z < s^*$. By assumption, $F'(s)s \geq F'(z)z$. Hence the second term in (16) is non-negative.

This proves that $G(z) - H(z) > 0$ for $s < z < s^*$. Now, when $s^* < z < S$, write

$$G(z) - H(z) = [F'(z)z - F'(S)S] \frac{F'(s)}{S} (\ln S - \ln s) + \left(\frac{F'(s)}{S} - \frac{F'(S)}{S}\right) F'(z)z(\ln z - \ln S).$$

...(23)

By assumption, $F'(z)z \geq F'(S)S$. Thus, in (17), the first term in brackets is non-negative. Since $F'(z) < 0$ for $s^* < z < S$, the second term is positive.

We have shown that $G(z) - H(z) > 0$ for all $s < z < S$. Hence, by (20) and (5')

$$\int_s^S G(z)z^{r/\theta - 1}dz > \int_s^S H(z)z^{r/\theta - 1}dz = 0$$

from which it follows, by (19). that $de/dg < 0$. ||

...(24)

In order to show that Condition (M) is not redundant, we provide in Section 8 an example which illustrates that if this condition is not satisfied then $de/dg$ may be positive.

4. DISCUSSION

The role of Condition (M) in establishing the effect of a change in $g$ on $e$ is best seen by re-examining the first-order conditions (4)-(5). Due to the recursive nature of the solution, these equations can be expressed in terms of the initial nominal price set by the firm, $p_1$, and the length of the interval, $\epsilon$, during which the price is held fixed.

$$g \int_0^\epsilon F'(p_1e^{-\theta t})p_1e^{-\theta t}dt - r\beta = 0$$

...(4')

$$\int_0^\epsilon F'(p_1e^{-\theta t})p_1e^{-(r+\theta)t}dt = 0.$$  

...(5')

Consider the effect of an increase in $g$ on $p_1$, holding $\epsilon$ constant. Let $p_1$ be adjusted so as to satisfy condition (5'). Using (M) we will show that after such an adjustment, the L.H.S. of (4') will be positive, which calls for a reduction in $\epsilon$.

Note first that under (M), an increase in $g$ raises, for given $p_1$ and $\epsilon$, the L.H.S. of (5'). To restore equality, $p_1$ must be increased. Thus, initially real price is higher. Due to (M), however, it must be the case that the new final real price is lower. This is illustrated in Figures 2 (a) and 2 (b).

These figures describe the real price and the effect of a change in $p_1$ on real profits as functions of time, for two alternative rates of inflation, $g_0$ and $g_1$ (> $g_0$). By (M), the curves in Figure 2 (b) must be non-decreasing in $t$. Furthermore, the difference between these curves has an opposite sign to the difference between the real price curves in Figure 2 (a). Hence if a higher rate of inflation leads to uniformly higher real prices, the curves in 2 (b) would not intersect, and condition (5') could not be satisfied.
By (5'), the discounted area under both curves in 2 (b) is equal to zero. Therefore, the undiscounted area under the marginal profit curve with the higher rate of inflation, \( g_1 \), must be larger than the corresponding area for the curve with \( g_0 \). Hence the L.H.S. of (4') must be higher for \( g_1 \). This means that the difference \( F(S) - F(s) \), which is the cost of postponing a price change, increases. It thus becomes profitable to reduce \( \varepsilon \) so as to satisfy this condition.

5. CHANGES IN ADJUSTMENT COSTS

To find the effects of a change in price adjustment costs, we differentiate (4')-(5') totally w.r.t. \( \beta \), which yields

\[
\frac{dS}{d\beta} = -\frac{r}{\Delta} F'(s)^{\sigma/\theta} > 0
\]

...(25)
\[ \frac{ds}{d \beta} = - \frac{r}{\Delta} F'(S)S^{r/\theta} < 0. \] \tag{26}

Hence, by definition
\[ \frac{de}{d \beta} = \frac{1}{g} \left( \frac{1}{S} \frac{ds}{d \beta} - \frac{1}{s} \frac{ds}{d \beta} \right) > 0. \] \tag{27}

As expected, higher adjustment costs lead to less frequent and to larger price changes.

6. CHANGES IN THE REAL RATE OF INTEREST

One expects a relation between the rate of inflation and the real rate of interest, depending on the effect of inflation on the level of real savings and investment. It is therefore of interest to study the effects of a change in the real rate of interest on the optimal price adjustment policy in the presence of inflation.

To find the effects of a change in \( r \), we differentiate (4')-(5') totally, yielding:
\[ \frac{dS}{dr} = \frac{-F'(s)}{g \Delta} \left[ \int_{s}^{S} F'(z)z^{r/\theta} \ln z dz + \beta g z^{r/\theta} \right] \] \tag{28}

substituting for \( \beta \) from (4') and integrating, using (5'),
\[ = \frac{-F'(s)}{g \Delta} \int_{s}^{S} \left[ F'(z)z^{r/\theta} \ln z + F(z) - F(S) \right] z^{(r/\theta)-1} dz. \]

Similarly,
\[ \frac{ds}{dr} = \frac{-F'(S)}{g \Delta} \left[ \int_{s}^{S} F'(z)z^{r/\theta} \ln z dz + \beta g S^{r/\theta} \right] \]
\[ = \frac{-F'(S)}{g \Delta} \int_{s}^{S} \left[ F'(z)z^{r/\theta} \ln z + F(z) + F(s) \right] z^{(r/\theta)-1} dz. \] \tag{29}

Observe first that, by (8) and (5') we have
\[ \frac{dV_1}{dr} = \frac{-1}{rg(S^{r/\theta} - S'^{r/\theta})} \int_{s}^{S} \left[ F'(z)z^{r/\theta} \ln z + F(z) \right] z^{(r/\theta)-1} dz. \] \tag{30}

Hence, by (10),
\[ V_1 + r \frac{dV_1}{dr} = \frac{-1}{g(S^{r/\theta} - S'^{r/\theta})} \int_{s}^{S} \left[ F'(z)z^{r/\theta} \ln z + F(z) - F(s) \right] z^{(r/\theta)-1} dz. \] \tag{31}

In view of (31), (29) may be rewritten
\[ \frac{ds}{dr} = \frac{1}{F'(s)} \left( V_1 + r \left( \frac{dV_1}{dr} \right) \right). \] \tag{32}

Also, from (31) and (4'), (28) may be rewritten
\[ \frac{dS}{dr} = \frac{1}{F'(S)} \left( V_1 + r \left( \frac{dV_1}{dr} + \beta \right) \right). \] \tag{33}

Thus, the sign of \( ds/dr \) is the same as the sign of \( V_1 + r(dV_1/dr) \), which is the change in \( r V_1 \) (the "permanent real profit") w.r.t. an increase in \( r \). Note that \( V_1 + r(dV_1/dr)<0 \) is a necessary condition for \( ds/dr = ((1/S)ds/dr - (1/s)ds/dr)/g > 0 \), because otherwise, by (32) and (33), \( ds/dr > 0 \) and \( dS/dr < 0 \). We shall now prove that this condition is always satisfied.
Proposition 3. $ds/dr < 0$ and $dS/dr < 0$.

Proof. Let $B(z) = \int_s^z F'(x)x^{r/g}dx$. Then, integrating by parts

$$\int_s^z [F'(z)z \ln z + F(z)]x^{r/g-1}dz = \int_s^z [F(z)x^{r/g}-B(z)]\frac{1}{z}dz. \quad \text{...}(34)$$

Integrating $B(z)$ by parts

$$B(z) = F(z)x^{r/g} - F(s)x^{r/g} - \frac{r}{g} \int_s^z F(x)x^{r/g-1}dx. \quad \text{...}(35)$$

Substituting (35) in (34),

$$\int_s^z [F'(z)z \ln z + F(z)]x^{r/g-1}dz = \int_s^z \left[ F(s)x^{r/g} + \frac{r}{g} \int_s^z F(x)x^{r/g-1}dx \right] \frac{1}{z}dz. \quad \text{...}(36)$$

From (29) and (36),

$$\frac{ds}{dr} = -\frac{F'(S)}{g\Delta} \int_s^z \left[ F(s)x^{r/g} - F(S)x^{r/g} \right] \frac{1}{z}dz \quad \text{...}(37)$$

$$= -\frac{rF'(S)}{g^2\Delta} \int_s^z \left[ F(S) - F(s) \right]x^{r/g-1}dx \frac{1}{z}dz < 0,$$ since $F(x) > F(s)$ for all $s < x \leq S$, $F'(S) < 0$ and $\Delta < 0$.

Now it follows from (36) and (28) that

$$\frac{dS}{dr} = -\frac{F'(S)}{g\Delta} \int_s^z \left[ F(s)x^{r/g} - F(S)x^{r/g} \right] \frac{1}{z}dz \quad \text{...}(38)$$

where

$$G(z) \equiv (F(s)-F(S))x^{r/g} + \frac{r}{g} \int_s^z (F(x)-F(S))x^{r/g-1}dx \quad \text{...}(39)$$

using (5'),

$$= \frac{r}{g} \left[ \int_s^z (F(x)-F(S))x^{r/g-1}dx - \int_s^z (F(x)-F(S))x^{r/g-1}dx \right]$$

$$= -\frac{r}{g} \int_s^z (F(x)-F(S))x^{r/g-1}dx.$$

Clearly, $G(S) = 0$. Also,

$$G(s) = -\frac{r}{g} \int_s^S (F(x)-F(S))x^{r/g-1}dx = (F(s)-F(S))s^{r/g} < 0. \quad \text{...}(40)$$

Furthermore,

$$G'(z) = \frac{r}{g} (F(z)-F(S))z^{r/g-1}, \quad \text{...}(41)$$

which changes sign once, being negative for small and positive for large values of $z$. It follows that $G(z) < 0$ for all $s \leq z < S$. Since $F'(s) > 0$ and $\Delta < 0$, (38) implies that $dS/dr < 0$.
Since over a typical cycle, real marginal profits are first negative and then positive, the net effect of an increase in the rate of interest is to reduce the present value of marginal profits. Therefore the firm's response is to increase the relative weight of the positive marginal profits by decreasing the upper and lower real prices. The sign of \( \frac{de}{dr} \) is in general ambiguous. In Section 8 an example is provided in which \( \frac{de}{dr} \) has opposite signs for alternative values of the parameters. However, we can provide a sufficient condition which determines the sign of \( \frac{de}{dr} \).

From (28)-(29) and the definition of \( e \), we have

\[
\frac{de}{dr} = \frac{1}{g^{2}\Delta s^{S}} \left[ F'(s)S - F'(s)S \right] \int_{S}^{S} F'(z)z^{\tau_{i}g} \ln zdz + \beta g(F'(S)S^{(\tau_{i}g)+1} - F'(s)S^{(\tau_{i}g)+1}) \right] \quad \ldots(42)
\]

using (4') and (5')

\[
= \frac{1}{g^{2}\Delta s^{S}} \left[ F'(s)S - F'(s)S \right] \int_{S}^{S} \frac{1}{z} \int_{S}^{S} F'(x)x^{\tau_{i}g}dx\ln zdz + \frac{g}{r} \int_{S}^{S} F'(x)dx(F'(S)S^{(\tau_{i}g)+1} - F'(s)S^{(\tau_{i}g)+1}) \right] \]

\[
= \frac{[F'(s)S - F'(s)S]}{g\Delta s^{S}} \beta \int_{S}^{S} \left[ \phi(z) - \psi(z) \right] z^{\tau_{i}g}dz
\]

where

\[
\phi(z) = \frac{r}{g} \frac{\int_{S}^{S} F'(x)x^{\tau_{i}g}dx}{\int_{S}^{S} F'(x)dx}, \quad \psi(z) = \frac{F'(z) + zF''(z)}{\int_{S}^{S} F'(z) + zF''(z)dz}. \quad \ldots(43)
\]

Using (5'), it can be verified that \( \int_{S}^{S} \phi(z)dz = \int_{S}^{S} \psi(z)dz = 1 \). Furthermore, \( \phi(z) \geq 0 \) and under Condition (M), \( \psi(z) \geq 0 \) for all \( s \leq z \leq S \), wherever they exist.

The sign of \( \frac{de}{dr} \) is thus seen to be determined by the difference in the expected value of the discount factor \( z^{\tau_{i}g} \) under the distributions \( \phi(z) \) and \( \psi(z) \). This fact suggests the application of a "risk-dominance" criterion (Hadar and Russell [4]). Condition (M) prevents a direct application of this criterion to expression (42) in its present form. However, a transformation of (42) into time units leads to the desired condition.

Let

\[
\hat{\phi}(z) \equiv g\phi(z)z \quad \text{and} \quad \hat{\psi}(z) = g\psi(z). \quad \ldots(44)
\]

Also, recall that \( z = p_{1}e^{-\delta t} \). We now state:

**Proposition 4.** If \( \hat{\phi}(z) \) is risk-dominant to \( \hat{\psi}(z) \) then \( \frac{de}{dr} > 0 \) and vice-versa.

**Proof.** Using the above transformation for \( z \), it is seen that \( \hat{\phi}(z) \geq 0, \hat{\psi}(z) \geq 0 \), and

\[
\int_{0}^{e} \hat{\phi}(z)dt = 1. \quad \text{Furthermore, (42) can be rewritten}
\]

\[
\frac{de}{dr} = \frac{[F'(s)S - F'(s)S]}{g\Delta s^{S}} \beta \int_{0}^{e} \left[ \hat{\phi}(z) - \hat{\psi}(z) \right] z^{\tau_{i}g}dt. \quad \ldots(45)
\]

Now, integrating by parts

\[
\int_{0}^{e} \left[ \hat{\phi}(z) - \hat{\psi}(z) \right] z^{\tau_{i}g}dt = r \int_{0}^{e} z^{\tau_{i}g} \int_{0}^{e} \left[ \hat{\phi}(x) - \hat{\psi}(x) \right] dxdt. \quad \ldots(46)
\]

Thus, a sufficient condition for \( \frac{de}{dr} > 0 \) is that \( \int_{0}^{t} \left[ \hat{\phi}(x) - \hat{\psi}(x) \right] dx < 0 \) for all \( 0 < t \leq e \).

That is, \( \hat{\phi} \) is risk-dominant to \( \hat{\psi} \). Clearly, \( \frac{de}{dr} < 0 \) if \( \hat{\psi} \) is risk-dominant to \( \hat{\phi} \).
It should be noted that risk-dominance is sufficient, but not necessary, to determine the sign of $de/dr$.

The appearance of distribution functions should not be surprising. Both the effect of a change in the rate of interest and the effect of a change in the initial nominal price are spread throughout the price cycle. The distributions $\phi$ and $\psi$ describe the change in $S$, holding $S/S$ (and thus $e$) constant, required to equilibrate conditions (4') and (5'), respectively. Their difference will determine the direction of the optimal change in $e$.

It is interesting to compare the difference between the effects of an increase in $\beta$ to the effects of an increase in $r$. Generally, one expects their effects to be similar. However, an increase in $r$ is seen to decrease prices unambiguously while an increase in $\beta$ raises initial prices and decreases terminal prices (thus increasing the amplitude of price changes). Similarly, the change in the length of the price cycle is ambiguous in the former case, and positive in the latter. These differences are due to the fact that an increase in $\beta$ increases only the benefits of further postponing a price change, while an increase in $r$ also reduces the marginal profitability of a nominal price increase.

7. THE EFFECTS OF TAXES

For policy purposes it is important to be able to predict the effects of excise taxes on the firm's price adjustment policy. In general, such taxes will affect both the level and the frequency of price changes.

Consider first a specific tax, levied at a fixed real level of $\theta \geq 0$ per unit sold. Real after tax profits, $G(z, \theta)$, are given by

$$G(z, \theta) = F(z) - \theta f(z),$$  \hspace{1cm} (47)

where $f(z)$ is the quantity demanded and $z$ is the real price facing the consumer. We assume that $G(z, \theta)$, as $F(z)$, is strictly quasi-concave and has a unique maximum. Given this assumption, the general nature of the solution remains unchanged. The first-order conditions for a maximum are now given by (7) with $G(z, \theta)$ replacing $F(z)$. Using the definition of $G(\cdot)$ these may be written

$$F(s) - F(S) + \theta [f(S) - f(s)] + r\beta = 0$$ \hspace{1cm} (48)

$$\int_s^S F'(z) - \theta f'(z) z^{r/\theta} dz = 0.$$ \hspace{1cm} (49)

Differentiating (48)-(49) w.r.t. $\theta$, we have

$$\frac{dS}{d\theta} = \frac{G_z(S, \theta)}{\Delta} \int_s^S f'(z)(z^{r/\theta} - S^{r/\theta}) dz > 0$$ \hspace{1cm} (50)

$$\frac{ds}{d\theta} = \frac{G_z(S, \theta)}{\Delta} \int_s^S f'(z)(z^{r/\theta} - S^{r/\theta}) dz > 0.$$ \hspace{1cm} (51)

where $\Delta = G_z(S, \theta)G_z(S, \theta)(S^{r/\theta} - S^{r/\theta}) < 0$, and $G_z(\cdot)$ is the partial derivative w.r.t. $z$. We assume a regularly shaped demand function with $f'(z) < 0$.

As one would expect, the effect of the tax is uniformly to increase the real price paid by consumers. The effects of the tax on the frequency of price adjustments depends on further assumptions. By (50), (51) and the definition of $e$, we have

$$\frac{de}{d\theta} = \frac{1}{g_{\Delta} z S} \int_s^S f'(z)[G_z(s, \theta)S(z^{r/\theta} - S^{r/\theta}) - G_z(S, \theta)S(z^{r/\theta} - S^{r/\theta})] dz$$

$$= \int_s^S f'(z) dz \int_s^S [G_z(z, \theta) + z G_{zz}(z, \theta)] dz$$

$$\frac{1}{g_{\Delta} S} \int_s^S [\phi(z) - \psi(z)] z^{r/\theta} dz,$$ \hspace{1cm} (52)
where

$$\phi(z) = \frac{G_x(z, \theta) + zG_{zz}(z, \theta)}{\int_s^{[G_x(z, \theta) + zG_{zz}(z, \theta)]dz}}, \quad \psi(z) = \frac{f'(z)}{\int_s^{f'(z)}dz}.$$  

...(53)

Clearly, \(\int_s^{\phi(z)}dz = \int_s^{\psi(z)}dz = 1\). Furthermore, under Condition (M) and the assumption \(f' < 0\), \(\phi(z) \geq 0\) and \(\psi(z) \geq 0\). Thus \(\phi(z)\) and \(\psi(z)\) can be regarded as density functions, and one may apply the results from the theory of "risk-dominance". Specifically, it can be seen from (52) that \(\delta e/d\theta\) has the opposite sign to

$$\int_s^{\phi(z)}[\phi(z) - \psi(z)]z^{r/\theta}dz = z^{r/\theta} \int_s^{[\phi(z) - \psi(z)]}dx - r \int_s^{g \int_s^{z^{r/\theta}}dz} \int_s^{[\phi(z) - \psi(z)]}dx dz$$

$$= - \frac{r}{g} \int_s^{z^{r/\theta}}dz \int_s^{[\phi(z) - \psi(z)]}dx dz.$$ 

...(54)

Hence \(\delta e/d\theta\) will be positive or negative depending upon whether \(\psi(z)\) is risk-dominant or risk-inferior to \(\phi(z)\).

As seen from (53), \(\phi(z)\) reflects the relative change in \(zG_x(z)\), while \(\psi(z)\) reflects the relative change in \(f(z)\), with respect to an increase in real price. In principle, their difference may go either way.

A similar analysis can be applied to the case of an *ad valorem* tax, levied as a fixed percentage of sales. If the tax rate is \(\theta\) \((0 \leq \theta < 1)\), then after tax real profits are given by \(G(x, \theta) = F(x) - \theta zf(x)\). The expressions for \(dS/d\theta\) and \(ds/d\theta\) are the same as (50) and (51) with \(f(z) + zf'(z)\) replacing \(f'(z)\). This term is the marginal revenue w.r.t. a price increase, and it is negative or positive depending upon whether demand elasticity is larger or smaller than unity. Unlike the standard monopoly case, the firm need not operate exclusively at prices associated with positive marginal revenue. Nevertheless, the first-order conditions impose a constraint on the "average" marginal revenue. Using the first-order conditions, equations (50) and (51) can be rewritten as:

$$\frac{dS}{d\theta} = \frac{G_x(S, \theta)}{\Delta(1-\theta)} \int_s^{m(z)(z^{r/\theta} - z^{r/\theta})dz} r\dot{S}^{r/\theta}{\theta}$$ 

...(55)

$$\frac{ds}{d\theta} = \frac{G_x(S, \theta)}{\Delta(1-\theta)} \int_s^{m(z)(z^{r/\theta} - z^{r/\theta})dz} r\dot{S}^{r/\theta}{\theta},$$ 

...(56)

where \(m(z) = d[c(f(z), f'(z)]/dz\) is the marginal cost w.r.t. a price increase. Clearly \(m(z) < 0\) since \(f'(z) < 0\) and marginal costs w.r.t. output increases can be assumed to be positive. It is seen that \(dS/d\theta > 0\). As in the case of a specific tax, the firm increases the initial real price as a result of a tax increase. However, the sign of \(ds/d\theta\) as well as \(de/d\theta\) appear to be ambiguous. The source of the difference between the results in these two cases is that with an *ad valorem* tax, tax payments increase with revenue, which provides an incentive for the firm to allow some reduction in prices.

8. SOME NUMERICAL EXAMPLES

Our purpose in this section is to provide counter examples to two presumably intuitive notions. First, that in the presence of price adjustment costs, an increase in the expected rate of inflation will lead to a higher frequency of price changes. Second, that an increase in
the real rate of interest will lead to a lower frequency of price changes. The examples can also provide some insight to the potential importance of price adjustment costs.

Suppose that $F(z)$ is piecewise linear:

$$ F(z) = \begin{cases} 
  az, & z \leq s^* \\
  (a+b)s^* - bz, & z > s^* ,
\end{cases} \quad \quad \quad (57) $$

where $a > 0$ and $b > 0$ are constants. Conditions (4')-(5') are in this case

$$ -a(s^* - s) + b(S - s^*) + r\beta = 0 \quad \quad \quad (58) $$
$$ -b(S^{(r/\beta)+1} - s^*(r/\beta+1)) + a(S^{(r/\beta)+1} - s^*(r/\beta+1)) = 0. \quad \quad \quad (59) $$

Let the parameters be: $s^* = 1$, $a = 0.5$, $b = 15$, $r = 0.05$ and $\beta = 0.85$. Then the relation between alternative values of $s$ and $\varepsilon$ can be calculated from (58)-(59)

<table>
<thead>
<tr>
<th>Rate of inflation (%)</th>
<th>0.14</th>
<th>0.18</th>
<th>0.22</th>
<th>0.26</th>
<th>0.30</th>
<th>0.34</th>
<th>0.38</th>
<th>0.42</th>
<th>0.46</th>
<th>0.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency of price change (ε years)</td>
<td>7.16</td>
<td>6.66</td>
<td>6.37</td>
<td>6.21</td>
<td>6.15</td>
<td>6.19</td>
<td>6.33</td>
<td>6.61</td>
<td>7.09</td>
<td>8.03</td>
</tr>
</tbody>
</table>

It is seen that at inflation rates above 30 per cent, increases in the inflation rate lead to an increase in $\varepsilon$, i.e. to a reduction in the frequency of price change. Notice that the function (57) violates Condition (M) for $z < s^*$. Accordingly, the example was chosen with a negatively skewed function ($b > a$), so as to increase the length of time spent in this region. For the same purpose, the adjustment costs were set at relatively high level (higher than the maximum profit flow), which explains the unrealistically long intervals between price changes.

This counter example may suggest that instances of reduced frequency of price changes, though theoretically possible, are not likely to be met in practice. For short intervals, the firm is in the neighbourhood of $s^*$, where Condition (M) is satisfied.

The profit function (57) can also be used to show that increases in the rate of interest may increase $\varepsilon$, i.e. increase the frequency of price changes.

Let $s^* = 1$, $a = 0.5$, $b = 0.001$, $g = 0.1$ and $\beta = 0.0001$. Then the relation between $r$ and $\varepsilon$ is:

<table>
<thead>
<tr>
<th>Rate of interest</th>
<th>0.04</th>
<th>0.06</th>
<th>0.08</th>
<th>0.10</th>
<th>0.12</th>
<th>0.14</th>
<th>0.16</th>
<th>0.18</th>
<th>0.20</th>
<th>0.30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency of price change (ε years)</td>
<td>1.341</td>
<td>1.336</td>
<td>1.330</td>
<td>1.324</td>
<td>1.314</td>
<td>1.313</td>
<td>1.308</td>
<td>1.302</td>
<td>1.297</td>
<td>1.270</td>
</tr>
</tbody>
</table>

It is seen that the "anomalous" case occurs for fairly realistic intervals between price changes. It must be noted, however, that a high degree of positive skewness, with the firm spending most of the time at the range of negative marginal profits, was required to produce the example.

Finally, we would like to report that numerical experiments with a quadratic profit function (which, in the absence of production costs, is obtained from a linear demand function) give high intervals between price changes (1-2 years) even with very low adjustment costs. This is due to the fact that, unlike (57), the loss in profits from any price $z \neq s^*$ in the neighbourhood of $s^*$, is relatively small.

9. CONCLUDING REMARKS

In the presence of costs of adjustment, even a fully anticipated inflation entails a real cost. For the firm this is reflected in a reduction in real discounted profits as the rate of inflation
increases. However, we would like to inquire whether the reduction in profits is also a social cost to the economy. The question arises since the monopoly tends to choose lower real prices (and hence larger quantities) over the last part of each period in which the nominal price is fixed, which may increase net social welfare.

A standard criterion which may be applicable in the present partial equilibrium context is that of consumer's surplus. Thus, let \( G(z) \), be net consumer's surplus associated with a real price \( z \),

\[
G(z) = F(z) + \int_{z}^{\infty} f(z) dz.
\]  \ ...(60)

The socially optimal policy is defined as the sequence of nominal prices and dates that maximizes (7) with \( G(z) \) replacing \( F(z) \) in the objective function. If we assume that \( G, \) as \( F, \) is strictly quasi-concave, then the socially optimum policy will have the same qualitative properties as the monopoly's optimum policy, the difference being that real prices vary around the competitive price which is equal to real marginal costs.

A natural measure for the "social costs of inflation" would be the difference between the discounted value of consumer's surplus in the absence of inflation and the discounted value of consumer's surplus minus the costs of price adjustment in the presence of inflation. This difference can be evaluated under alternative market organizations. In particular, it may be evaluated under the socially optimal \((s, S)\) policy and under the monopoly \((s, S)\) policy.

Another question of interest concerns the measure of monopoly costs under inflation. This would involve the difference in the level of the social welfare function under the socially optimal and under the monopoly's \((s, S)\) policies for a given expected rate of inflation. One would like to investigate the effect of changes in the expected rate of inflation on this measure.

Two other extensions seem to be of interest. First would be the introduction of inventory decisions on behalf of the firm and the consumers. For example, if consumers can store the commodity, then price differences cannot exceed storage costs. Another application of the analysis is to input decisions, particularly with respect to the duration of wage contracts, since changes in wage contracts may also entail real costs.

The analysis can also be extended to a multi-product firm. A natural question is whether the optimal frequency of price changes will be uniform across products. Returns to scale in the provision of information may lead the firm to announce all price changes simultaneously.

Finally, some macroeconomic implications of the analysis can be noted. According to the analysis, with a constant rate of inflation, prices will increase on the average at the rate of inflation. More importantly, if the timing of firms' price adjustments is independent, then we would observe a variance of price changes across products or firms which increases with the rate of inflation. This implies that informational costs exist even with a steady aggregate rate of inflation. Furthermore, if, as the analysis suggests, firms adjust prices more frequently as the rate of inflation increases unexpectedly, then one would expect to observe a larger initial adjustment in the average price the larger the rate of inflation.

Price adjustment policies of the kind described in this paper are particularly visible with respect to the rate of exchange. Differences in rates of inflation on the one hand and informational costs of adjustment on the other hand induce many countries to engage in periodic exchange rate adjustments. Our analysis could be applicable to the analysis of optimal rate of exchange policies.

Costs of adjustment have recently become a central issue in taxation policies. Inflation continuously erodes the basis of various taxes (such as the property tax). In view of the considerable costs of asset revaluation, standard practice is periodic reassessment. It would be of interest to investigate the dependence of the optimal frequency of such revaluations and of the optimal tax rates on the expected rate of inflation.
APPENDIX

The firm looks for a pair of sequences \( \{t^*_t\} \) and \( \{p^*_t\} \), \( \tau = 1, 2, 3, \ldots \), that maximize the present discounted value of real profits at time \( t_0 = 0 \):

\[
V_0 = \sum_{t = 0}^{\infty} \left[ \int_{t_t}^{t_{t+1}} F(p_t e^{-\eta t}) e^{-\eta t} dt - \beta e^{-r(t_{t+1} - t_t)} \right]. \tag{A1}
\]

Denote an optimal pair of such sequences by \( \{t^*_t\} \) and \( \{p^*_t\} \). We assume that such solutions exist with \( V_0 \geq 0 \) at the optimum (since no production is always feasible). We now have the following:

**Theorem.** For any solution \( \{t^*_t\} \) and \( \{p^*_t\} \), there exists a unique \( \varepsilon > 0 \), such that \( t^*_{t+1} = t^*_t + \varepsilon \) and \( p^*_{t+1} = p^*_t e^{\eta \varepsilon} \).

**Proof.** By the principle of optimality we know that if \( \{t^*_t\} \) and \( \{p^*_t\} \) maximize \( V_0 \), they also maximize the discounted real profits at the points of time \( t^*_1, t^*_2, \ldots \), and so on, which we denote \( V_1, V_2, \ldots \). Specifically,

\[
V_1 = \sum_{t = 1}^{\infty} \left[ \int_{t_t}^{t_{t+1}} F(p_t e^{-\eta t}) e^{-\eta t} dt - \beta e^{-r(t_{t+1} - t_t)} \right] \tag{A2}
\]

\[
V_2 = \sum_{t = 2}^{\infty} \left[ \int_{t_t}^{t_{t+1}} F(p_t e^{-\eta t}) e^{-\eta (t_t - t_{t+1})} dt - \beta e^{-r(t_{t+1} - t_t)} \right] \tag{A3}
\]

changing the summation index

\[
= \sum_{t = 1}^{\infty} \left[ \int_{t_t}^{t_{t+2}} F(p_{t+1} e^{-\eta t}) e^{-\eta (t-t_{t+1})} dt - \beta e^{-r(t_{t+2} - t_t)} \right].
\]

Using the transformation \( u = t - t^*_1 \), rewrite (A2)

\[
V_1 = \sum_{t = 1}^{\infty} \left[ \int_{t_t - t^*_1}^{t_{t+1} - t^*_1} F(p_t e^{-\eta t} e^{-\eta u}) e^{-\eta u} du - \beta e^{-r(t_{t+1} - t_t)} \right]. \tag{A4}
\]

Similarly, using \( u = t - t^*_2 \), rewrite (A3) as

\[
V_2 = \sum_{t = 1}^{\infty} \left[ \int_{t_t - t^*_2}^{t_{t+2} - t^*_2} F(p_{t+1} e^{-\eta t} e^{-\eta u}) e^{-\eta u} du - \beta e^{-r(t_{t+2} - t_t)} \right]. \tag{A5}
\]

We observe that the variables to be determined in \( V_1 \) are

\[
\{t_t - t^*_1\} = \{0, t_2 - t^*_1, t_3 - t^*_1, \ldots\} \quad \text{and} \quad \{p_t e^{-\eta t^*_1}\} = \{p^*_1 e^{-\eta t^*_1}, p^*_2 e^{-\eta t^*_1}, \ldots\}.
\]

In \( V_2 \) they are \( \{t_{t+1} - t^*_2\} = \{0, t_3 - t^*_2, t_4 - t^*_2, \ldots\} \) and \( \{p_{t+1} e^{-\eta t^*_2}\} = \{p^*_2 e^{-\eta t^*_2}, p^*_3 e^{-\eta t^*_2}, \ldots\} \).

The functions \( V_1 \) and \( V_2 \) are identical in the corresponding variables. Hence

\[
t^*_2 - t^*_1 = t^*_3 - t^*_2 = \ldots = \varepsilon
\]

and

\[
p^*_1 e^{-\eta t^*_1} = p^*_2 e^{-\eta t^*_2} = \ldots
\]

or

\[
p^*_2 = p^*_1 e^{\eta (t^*_2 - t^*_1)} = p^*_1 e^{\eta \varepsilon}, \text{ etc.}
\]

We have shown that any solution to (A.1) has a recursive form. In the text we show that the recursive solution is unique.
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NOTES

1. This formulation assumes that output is always identical to sales. It was pointed out to us by Edi Karni that in the presence of costs of price adjustment a monopoly may prefer not to satisfy demand. The general definition of real profits should be:

$$\text{F}(z_i) = \max \{q_i \leq r_i; \{[z_i - c(q_i)]q_i\} \}.$$  

A sufficient condition for the maximum to be attained at $q_i = f(z_i)$ is that $c(q_i)$ be independent of $q_i$.

2. Clearly, if $F$ were non-negative for all $z \geq 0$, a policy of no price change would ensure $V_0 > 0$. This, however, does not seem to be an acceptable assumption.

3. Note that the definition of $V_1$ includes the cost of the first price change. To simplify notation, and without loss of generality, we assume hereon that $t_1 = 0$, so that $p_1 = p_1$.

4. Conditions (9)-(10) can be replaced by the equivalent conditions

$$F(S) - F(s) - r\beta = 0$$  

and (10). For any $S$, if there exists an $s$, such that $0 \leq s < S$, which satisfies (9'), this solution must be unique. This follows from the strict quasi-concavity of $F$. Furthermore, these solutions define a differentiable function $s = g(S)$, over an interval of $S$ (which obviously contains all the possible solutions). Consider the function $H(S) = V_1(s, S) = V_1(g(S), S)$, which is continuously differentiable in $S$. Clearly $H'(S) = 0$ is a necessary condition for an optimum and is equivalent to conditions (9)-(10). To show that the maximum is unique, it is sufficient that $H''(S) = 0 \Rightarrow H''(S) < 0$. Now, $H''(S)$ evaluated at the optimum has the opposite sign of the Jacobian of (9)-(10) which, by (11) is positive.

5. If $F(z)$ is twice differentiable, Condition (M) is equivalent to $F''(s) + F'(s)z < 0$ for all $z$. This is clearly stronger than the assumption that $F(z)$ be strictly concave, i.e. $F''(z) < 0$.

6. Note that a term $[(F'(s)/S - F'(s)/S)(F'(s)/S)]$ in $S$ has been added in the definition of $G(z)$. But this addition to (14) is, in view of (5'), equal to zero.

7. If production costs where zero $(c = 0)$, $F(z) = zf(z)$. Thus, (57) would imply that

$$f(z) = \begin{cases} 
\frac{a}{z} & \text{if } z \leq s^* \\
\frac{(a+b)s}{z} - b & \text{if } z > s^*
\end{cases}$$

i.e. the quantity demanded increases as the real price decreases to $s^*$, while any further price decrease leaves the quantity demanded unchanged.

8. Strictly speaking, this is true only for functions which are continuously differentiable.

9. In fact, more than non-storability is required. In our model demand depends only on the current price charged by the firm. It is assumed that while consumers have firm expectations with respect to the general price level, they ignore (due, presumably, to informational costs) the time pattern of future prices of any particular commodity. If consumers could in fact predict all prices with perfect foresight, they would obviously go on a buying spree just before the anticipated price increase. More generally, if substitution across time is admitted, demand will depend on the whole price cycle of each firm.

REFERENCES


